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ABSTRACT

Firms that supply goods to the government often produce these goods in conjunction with other goods, incurring joint or common production costs. When the government uses costs as a basis for contracting with such firms, questions of cost allocation naturally arise. This paper presents, in the context of a bidding model, some conditions under which a fixed-price contract is optimal (in the class of linear contracts) for the government. That is we give conditions under which the problem of cost allocation is totally avoidable.

I. INTRODUCTION

Firms that supply goods to the government often produce these goods in conjunction with other goods, incurring joint or common production costs. When the government uses cost as a basis for contracting with such firms, questions of cost allocation naturally arise. This paper presents, in the context of a bidding model, some conditions under which a fixed-price contract is optimal (in the class of linear contracts) for the government. That is, we give conditions under which the government is best off when the problem of cost allocation is totally avoidable.

McAfee and McMillan [1986], hereafter MM, analyze common linear contracts in a principal-agent framework that handles problems of adverse selection (selecting the firm with the lowest expected costs) and moral hazard (motivating the winning firm to expend effort to reduce actual costs). They compare fixed-price contracts, cost-plus contracts, and incentive contracts. With a cost-plus contract the government agrees to pay the contractor full costs plus a fixed fee (or fixed percentage of actual costs). Using a fixed-price contract the supplier agrees to deliver goods or services for an amount agreed upon before actual costs are realized. Finally, with an incentive contract the government agrees to a certain price (full estimated costs plus a fee) and to share in any deviation between actual and estimated costs.

MM first show that as long as there are two or more bidders a cost-plus contract is inferior to a fixed-price or an incentive contract. MM further demonstrate that for any finite number of bidders greater than or equal to two, if bidders expect rivals to face different costs, the government will be strictly better off with an incentive contract than with a fixed-price contract. The advantage of an incentive contract over a fixed-price contract, even when all bidders are risk neutral, is due to what MM refer to as a "bidding-competition effect", and is associated with the adverse selection problem of finding the low cost producer. Laffont and Tirole [1987] and McAfee and McMillan [1987] show that when all bidders are risk-neutral and when observed costs are as defined by MM, the optimal contract is indeed linear in observed costs.

Using a generalization of the MM model with risk-neutral bidders, we demonstrate that fixed-price contracts can strictly dominate incentive contracts. The apparent contradiction between our result and that of MM is due to the different interpretations of ex post observable cost. MM assume that the government can write and enforce contracts based on opportunity costs; i.e., they assume that ex post observable costs represent opportunity costs. In contrast, we assume that the government can only contract on observed costs; these observed costs include an allocated share of joint costs and differ from opportunity costs. This distortion between observable costs and opportunity costs is related to the effect of joint costs. Thus, the distortions caused by joint costs may be sufficient to vitiate the bidding-competition advantage associated with incentive contracts over fixed-price contracts. Note that McAfee and McMillan remark that their results may well depend on accounting costs being "...a close approximation to opportunity cost." [Footnote 16, p. 335].

In the next section, we present our model. In Section III, we demonstrate conditions under which a fixed-price contract dominates an incentive contract. We conclude with a brief summary in Section IV.

II. THE MODEL

Consider the case in which $n \geq 2$ risk-neutral agents, indexed $i=1,2,..,n$ have the ability to produce a product or provide a service specified by the government. Also, suppose that each agent currently produces and sells another product or service for the private sector. Let G be a measure of the output produced for the government, and let Q be a measure of (physically distinct) output produced for sale in the private sector.[1]

Following MM, we employ an independent-private-values model (Milgrom and Weber [1982]) in which agents differ only with respect to a productivity parameter, $V_i \in [V_*, V^*]$. The ith agent knows the value of the parameter V_i. All other agents $j \neq i$ and the principal (the government) know that V_i represents an independent draw from the distribution of V_i. We denote this distribution by $F(V)$, and the corresponding density by $f(V)$, where $F(V_*) = 0$ and $F(V^*) = 1$. Agent i's cost of producing Q and G is made up of separable cost in Q, $C_Q(Q)$, separable cost in G, $C_G(G)$, joint cost, $C_J(Q,G)$ and the agent's dollar cost of supplying unobservable effort, $h(e)$. The $h(e)$ function is twice continuously differentiable, strictly increasing and strictly convex; $C_Q(Q)$ is twice continuously differentiable, non-decreasing, and weakly convex; and $C_J(Q,G)$ is twice continuously differentiable and non-decreasing in Q. The agent's unobservable effort level is assumed to reduce the joint cost of production. Agent i's cost of producing Q and G with effort level e is then:

$$C_Q(Q) + C_G(G) + (V_i + W - e)C_j(Q,G) + h(e), \tag{1}$$

where W is a random cost factor that is common to all agents, is independent of V, and is such that $E[W] = 0$. Since the agents are all assumed to be risk-neutral, we can without loss of generality deal solely with the expected value of (1):

$$C_Q(Q) + C_G(G) + (V_i - e)C_J(Q,G) + h(e). \tag{2}$$

Purely fixed costs are assumed to be included in the joint cost term, so that $C_Q(0)=C_G(0)=0$.

Suppose that the government uses a first-price, sealed bid auction to award a contract for the production of a fixed quantity, G, of (customized) goods. Let b represent the winning bid, and let $a \in [0,1]$ represent the cost share ratio, and suppose that the contract calls for the government to pay the successful bidder:

$$b + a(C^*(Q,G) - b), \tag{3}$$

where $C^*(Q,G)$ is the bidder's (ex post) observable cost of meeting the contract, defined below. When $a = 0$, the contract is a fixed-price contract; when $a > 0$, the contract is an incentive contract.

In order to implement this incentive contract, the government and bidders must agree on a definition of observable cost. We assume that, without auditing costs, the government can determine ex post separable costs, $C_G(G)$, and ex post joint costs, $(V_i - e)C_J(Q,G)$.[2] The problem of defining observable costs then hinges around an allocation of joint costs between government output and other output. Let k(Q,G) be the allocation (function) rule, agreed upon by bidders and the government, that represents the share of joint costs assigned to the government contract. In general, a cost allocation k will have the following characteristics:

(i) $0 \leq k(Q,G) \leq 1$;

(ii) $k(Q,0) = 0$ for all positive Q;

(iii) $k(0,G) = 1$ for all positive G; and

(iv) $k(Q,G)$ is decreasing in Q, for all $G > 0$.

One common class of cost allocations may be written as $k(Q,G) = G/(Q+G)$. With this class of cost allocations the definition of output becomes critical. Thus, the joint costs to be allocated to a contract for 100 jet fighter planes depends on whether output is to be measured in terms of planes, labor hours, labor dollars, or some other measure.

Given a cost allocation rule, $k(Q,G)$, we may now write observable cost as:

$$C^*(Q,G) \equiv C_G(G) + k(Q,G)(V - e)C_J(Q,G). \tag{4}$$

Recall that, by definition, $h(e)$ is not observable and may not be charged to the project. Note that the existence of joint costs will generally cause observable costs to differ from opportunity cost for two reasons. First, observable costs do not represent incremental production cost of producing G. Second, each bidder would change the production level Q in response to being awarded the contract, so revenues from sale of the outside good, as well as production costs of that good, change.

Let $R(Q)$ be the revenue curve faced by agent i in the outside market where $R(\cdot)$ is twice continuously differentiable, weakly concave, and $R'(0) > 0$. We assume that each agent faces the same revenue curve for the sale of its private sector product. Note that we allow for the possibility that all bidders face a competitive outside market. However, the agents may all be selling different products in the outside markets. The assumption of common revenue functions enables us to use an independent-private-values model, and thus focus directly on the effect of joint costs on bidding for the government contract.

Before presenting the bidding optimization problem of potential agents, we introduce some additional notation. Let $P^*(V_i)$ be the profits of the ith bidder if the agent is not awarded the contract. Then:

$$P^*(V_i) = \{\max_{Q,e} R(Q) - C_Q(Q) - (V - e)C_J(Q,0) - h(e)\}. \tag{5}$$

Given a cost allocation rule $k(Q,G)$ and an incentive share ratio a, the profits of the ith bidder, if successful in winning the contract, are:

$$\begin{aligned}\hat{P}(b_i,V_i;a) = \max_{Q,e} \{&R(Q) - C_Q(Q) - C_G(G) - (V_i-e)C_J(Q,G) \\ &- h(e) + b_i + a[C^*(Q,G) - b_i]\}.\end{aligned} \tag{6}$$

Given our definition of $C^*(Q,G)$, we can write the winning bidder's maximum profit gross of the net bid payment, $(1 - a)b_i$, as

$$\begin{aligned}P(V_i;a) \equiv \max_{Q,e} \{&R(Q) - C_Q(Q) - (1 - a)C_G(G) \\ &- (1 - ak(Q,G))(V_i - e)C_J(Q,G) - h(e)\}.\end{aligned} \tag{7}$$

Denote the solution to (7) by $(Q(V_i,G;a),e(V_i,G;a))$. Then using (4) and (7) we can rewrite (6) as:

$$\hat{P}(b_i,V_i;a) = (1 - a)b_i + P(V_i;a). \tag{8}$$

Finally, we can define the economic cost of fulfilling the contract:[3]

$$C(V_i;a) \equiv P^*(V_i) - P(V_i;a). \tag{9}$$

Note that $C(V_i;a)$ is not required to be positive for all V_i. We can thus rewrite (8), the incremental profit for the winning bidder, as

$$\hat{P}(b_i,V_i;a) - P^*(V_i) = (1 - a)b_i - C(V_i;a). \tag{10}$$

When there are no joint costs of production, our model reduces to the MM model with risk-neutral bidders. In that case, $C_J(Q,0) = 0$ and $C_J(Q,G)$ for $G > 0$ can be taken as identically equal to one (or any constant). Then the expression in (9) can easily be shown to be equal to $(1 - a)[V_i + C_G(G) - e(V_i,G;a)] - h(e(V_i,G;a)$. The quantity $V_i + C_G(G)$ is what MM refer to as the "opportunity cost" for the ith agent, so that (9) is still the expected cost of the contract to a bidder net of the cost of effort.

Each agent i chooses his or her bid b_i to maximize the probability of winning times the expression in (10). The winning bidder is chosen via a first-price, sealed bid auction; thus, the probability of agent i winning is just the probability that $b_i < b_j$ for all $j \neq i$. A symmetric

Nash equilibrium bidding strategy is then a bid function $B(V_i;a)$ such that if $b_j = B(V_j;a)$ for all $j \neq i$, then $b_i = B(V_i;a)$ is the optimal bid for agent i. The probability that agent i is the winning bidder is then $[1 - F(B^{-1}(b))]^{(n-1)}$ since the b_j are all independent draws from the distribution of V. The optimal bid function $B(\cdot)$ is the solution to the following maximization problem, where subscripts have been deleted:

$$\max_b \; [1 - F(B^{-1}(b))]^{(n-1)}[(1-a)b - C(V;a)]. \tag{11}$$

It is straightforward to verify that B(V;a) satisfies the following first-order differential equation:

$$s(V)[(1-a)B(V;a) - C(V;a)] + (1-a)B'(V;a) = 0, \tag{12}$$

where $s(V) \equiv -(n-1)f(V)/(1-F(V))$. Let

$$S(V) \equiv \int_{V_*}^{V} s(t)dt \tag{13}$$

Then we can solve for $(1 - a)B(V;a)$ as[4]

$$(1 - a)B(V;a) = -\exp(-S(V))\int_{V}^{V^*} s(t)\exp(S(t))C(t;a)dt \tag{14}$$

The first thing that we must demonstrate is that B(V;a) is increasing in V, so that the contract is awarded to the most efficient bidder. From (12), and the fact that $s(V) < 0$ for all V, we see that

$B'(V;a)$ has the same sign as the net benefit of the contract to the winning bidder (equation (10)). Clearly, this net benefit must be non-negative in equilibrium. If it were equal to zero in equilibrium for some V, then the value of equation (11) would be zero. The bidder could improve his or her position (i.e. make the value of the maximand in (11) strictly positive) by increasing the bid, which makes the profit positive and decreases the probability of winning; however, it could not have been the case that the original bid was an equilibrium bid if this were possible. Therefore, it must be true that the net benefit of winning the contract is strictly positive in equilibrium. This results in $B'(V;a)$ strictly positive, and we have bids increasing in V.

The observed costs $C^*(\cdot)$ can be written in equilibrium as a function of V and the cost share parameter a. For simplicity, we retain the use of $C^*(\cdot)$ and define

$$C^*(V;a) \equiv C_G(G) + k(Q(V,G;a),G)(V - e(V,G;a))C_J(Q(V,G;a),G). \quad (15)$$

Given that we have demonstrated that the most efficient (lowest cost) bidder wins the contract, we can now compare fixed price contracts with incentive contracts. In the next section, we give sufficient conditions for the government (principal) to be strictly better off with a fixed-price contract in which the winning bidder bears all of the ex post cost of the contract.

III. OPTIMAL FIXED-PRICE CONTRACTS

The objective of the government is the **ex ante** minimization of its expenditures on the contract. If a bidder with cost parameter $V \in [V_*, V^*]$ is the winner, the government's conditional expected expenditure, where expectation is taken with respect to W and conditioned on V, will be

$$(1 - a)B(V;a) + aC^*(V;a). \tag{16}$$

The expected value of the expression in (16) (where expectation is now taken with respect to the distribution of the minimum of n independent draws from the distribution of V), is the **ex ante** expected expenditure for the government for a given cost share a.

We would like to determine conditions under which the expected expenditure is minimized at $a = 0$. _If_ the expression is (16) is minimized at $a = 0$ for _all_ V, then we are done. We proceed as follows: we first develop the general conditions under which (16) will be minimized at $a = 0$ for all V; we then give conditions on the underlying structure of our model (the revenue and cost functions) that are sufficient for these general conditions.

THEOREM 1

If observed costs are decreasing in the cost parameter V in equilibrium, then a fixed price contract strictly dominates an incentive contract. That is, if $C^*(V;a)$ is decreasing in V for all $V \in [V_*, V^*]$, then $(1 - a)B(V;a) + aC^*(V;a)$ is minimized at $a = 0$ for all $V \in [V_*, V^*]$.

Proof:

To demonstrate that $(1 - a)B(V;a) + aC^*(V;a)$ is minimized at $a = 0$, we show that

$$B(V;0) - (1 - a)B(V;a) - aC^*(V;a) < 0 \qquad \text{for all } 0 < a \leq 1. \tag{17}$$

From (14) we have

$$(1 - a)B(V;a) = -\exp(-S(V))\int_V^{V^*} s(t)\exp(S(t))C(t;a)dt$$

so that the left hand side of (17) can be rewritten as

$$-\exp(-S(V))\int_V^{V^*} s(t)\exp(S(t)[C(t;0) - C(t;a)]dt - aC^*(V;a). \tag{18}$$

We first demonstrate that

$$C(t;0) - C(t;a) < aC^*(t;a) \qquad \text{for all } t. \tag{19}$$

Using the definitions of $C(t;a)$ (from (9)), we have

$$\begin{aligned} C(t;0) - C(t;a) &= P^*(t) - P(t;0) - P^*(t) + P(t;a) \\ &= P(t;a) - P(t;0). \end{aligned} \tag{20}$$

The last expression in (20) is the difference between the firm's maximum profit when costs are shared and when they are not. From equation (7) (the definition of $P(t;a)$); the definition of $(Q(t,G;a),e(t,G;a))$ as the solution to the problem in (7); and equation (15) (the definition of $C^*(t;a)$) we have

$$P(t;a) - P(t;0) =$$

$$[\underset{Q,e}{\text{MAX}} \{R(Q)-C_Q(Q)-(1-a)C_G(G)-(1-ak(Q,G))(t-e)C_J(Q,G)-h(e)\}$$

$$- \underset{Q,e}{\text{MAX}} \{R(Q)-C_Q(Q)-C_G(G)-(t-e)C_J(Q,G)-h(e)\}] \quad <$$

$$[\underset{Q,e}{\text{MAX}} \{R(Q)-C_Q(Q)-C_G(G)-(t-e)C_J(Q,G)\} \tag{21}$$

$$+ aC_G(G) + ak(Q(t,G;a),G)(t - e)C_J(Q(t,G;a),G)$$

$$- \underset{Q,e}{\text{MAX}} \{R(Q)-C_Q(Q)-C_G(G)-(t-e)C_J(Q,G)-h(e)\}] \quad =$$

$$aC^*(t;a).$$

Therefore, we have our desired result that

$$C(t;0) - C(t;a) < aC^*(t;a).$$

Equations (18), (19), the left hand side of (17), and $s(t) < 0$ for all $t \in [V_*,V^*]$ yield

$$B(V;0) - (1 - a)B(V;a) - aC^*(V;a) \quad <$$

$$- \exp(-S(V))\int_V^{V^*} s(t)\exp(S(t))aC^*(t;a)dt - aC^*(V;a) \tag{22}$$

The major part of our proof demonstrates that the right hand side of

(22) is non-positive. We note that the right hand side of (22) has the same sign as

$$- \int_{V}^{V^*} s(t)\exp(S(t))aC^*(t;a)dt - \exp(S(V))aC^*(V;a) \tag{23}$$

If the expression in (23) is increasing in V and is less than or equal to 0 at V^*, then (23) is non-positive for all V. In addition, at $V = V^*$, the right hand side of (22) can be shown to be strictly negative.

To show (23) is increasing in V, we differentiate (23) with respect to V using Liebnitz's Rule:

$$s(V)\exp(S(V))aC^*(V;a) - s(V)\exp(S(V))aC^*(V;a) - \exp(S(V))\frac{\partial C^*(V;a)}{\partial V} = - \exp(S(V))\frac{\partial C^*(V;a)}{\partial V}. \tag{24}$$

since by definition $S'(V) = s(V)$. When $\frac{\partial C^*}{\partial V} < 0$ for all V, the right hand side of (24) is positive for all V, and we have (23) increasing in V. At $V = V^*$, (23) reduces to

$$- \exp(S(V^*))aC^*(V^*;a) = 0 \tag{25}$$

since $S(V^*) = - \infty$ by definition.

Therefore, (23) is less than or equal to zero. We now must show that the right hand side of (22) is less than zero at $V = V^*$. Taking the limit as $V \to V^*$, and applying L'Hopital's rule, we get

$$\lim_{V \to V^*} \left\{ \frac{- \int_V^{V^*} s(t)\exp(S(t))aC^*(t;a)dt}{\exp(S(V))} - aC^*(V;a) \right\} =$$

$$\lim_{V \to V^*} \left\{ \frac{s(v)\exp(S(V))aC^*(V;a)}{s(v)\exp(S(V))} - aC^*(V;a) \right\} = \tag{26}$$

$$\lim_{V \to V^*} \left\{ aC^*(V;a) - aC^*(V;a) \right\} =$$

$$0$$

We therefore have our desired result that

$$B(V;0) - (1 - a)B(V;a) - aC^*(V;a) < 0 \qquad \text{for all } V \in [V_*, V^*]. \tag{27}$$

Q.E.D.

The conditions of Theorem 1 state that even though the cost parameter V is increasing, which increases joint costs for all Q and G, the level of observed costs decrease <u>in equilibrium</u>. This results because the agent has the ability to adjust both the outside output, Q and the level of cost reducing effort, e. In fact, under only our initial assumptions, both Q and e can be shown to be decreasing in V.

It should be noted that for the MM model, the conditions of Theorem 1 can never be met. In their model, $C^*(V;a)$ equals $(V + C_G(G) - e(V,G;a))$. Since $e(\cdot)$ maximizes $(1 - a)e - h(e)$ in the MM model, it is invariant with respect to V. Therefore, $C^*(V;a)$ is strictly increasing with respect to V.

We can now provide sufficient conditions so that the requirements of Theorem 1 are met. We assume that marginal revenue minus marginal separable cost of the outside output is a positive constant, and that the ratio of allocated joint costs to the marginal joint costs borne by the agent is increasing in Q.

THEOREM 2

If (i) $R'(Q) - C_Q'(Q) > 0$ for all Q;

(ii) $R''(Q) - C_Q''(Q) = 0$ for all Q; and

(iii) $$\frac{k(Q,G)C_J(Q,G)}{\dfrac{\partial(1 - ak(Q,G))C_J(Q,G)}{\partial Q}}$$ is increasing in Q, then

then $C^*(V;a)$ is decreasing in V for all V and for all $0 \leq a \leq 1$.

Proof:[5]

For ease of exposition we will (1) denote all first (second) derivatives of all functions with respect to Q, e and V by primes, since functions depend on only one of these arguments, and (2) write functions without explicit reference to their arguments. Differentiating (15) (the definition of $C^*(V;a)$) totally with respect to V and collecting terms, gives us the following in equilibrium:

$$C^{*\prime}(V;a) = Q'(V - e)[k'C_J + kC_J'] + (1 - e')kC_J. \tag{28}$$

The first order conditions for the winning agent's maximization problem are:

$$\begin{aligned} &R' - C_Q' - (V - e)[-ak'C_J - akC_J' + C_J] = 0 \\ &(1 - ak)C_J - h' = 0. \end{aligned} \tag{29}$$

Second order conditions for the winning agent's problem are that the principal minors of the following Hessian matrix alternate in sign:

$$H = \begin{bmatrix} R''-C_Q''-(V-e)[-ak''C_J-2ak'C_J'-akC_J''] & -ak'C_J-akC_J'+C_J' \\ & \\ -ak'C_J-akC_J'+C_J' & - h'' \end{bmatrix} \tag{30}$$

or equivalently,

$$\begin{aligned} &R'' - C_Q'' - (V - e)[ak''C_J - 2ak'C_J' - akC_J'' + C_J''] < 0, \\ &- h'' < 0, \text{ and} \\ &| H | > 0. \end{aligned} \tag{31}$$

Total differentiation of the conditions in (29) with respect to V gives us:

$$H \begin{bmatrix} Q' \\ e' \end{bmatrix} = \begin{bmatrix} -ak'C_J - akC_J' + C_J' \\ 0 \end{bmatrix} \tag{32}$$

Using Cramer's Rule, we can solve for Q' and e':

$$\begin{aligned} Q' &= -h''[C_J' - ak'C_J + akC_J']/|H| \\ e' &= -[C_J' - ak'C_J + akC_J']^2/|H| \end{aligned} \tag{33}$$

From C_J increasing in Q and k decreasing in Q for all G, we have $(1 - ak)C_J$ increasing in Q; this, along with the second order conditions, give us Q and e strictly decreasing in V. Substituting the explicit expression for $|H|$ and (33) into (28), and collecting terms gives:

$$\frac{\partial C^*}{\partial V} = \frac{-h''}{|H|}\{(V-e)(k'C_J+kC_J')(C_J'-ak'C_J-akC_J') + aC_J(R''-C_Q'-(V-e)(C_J''-ak''C_J-2ak'C_J'+akC_J''))\}. \tag{34}$$

When $R'' - C_Q''(Q) = 0$, the expression in (34) reduces to:

$$\frac{\partial C^*}{\partial V} = \frac{-h''(V-e)}{|H|}\{(k'C_J+kC_J')(C_J'-ak'C_J-akC_J') - aC_J(C_J''-ak''C_J-2ak'C_J'-akC_J'')\}. \tag{35}$$

Since $(1 - ak)C_J$ is increasing in Q, $R' - C_Q' > 0$ and the first order conditions give us $(V - e) > 0$ in equilibrium. To show C^* decreasing in

V, we thus need the term in brackets on the right hand side of (35) positive. This is immediate from (iii) in the statement of the Theorem.
Q.E.D.

A sufficient, but not necessary condition for (i) and (ii) in Theorem 2 to hold is that both the price of Q and the marginal separable cost of Q are constants, with the former larger than the latter. Condition (iii) in Theorem 2 will hold, for example, if $k(Q,G) = G/(Q+G)$ and $C_J(Q,G) = QG + F$, where F is fixed costs and $G^2 > F$. The conditions in Theorem 2 can thus be met by reasonable revenue, separable cost, joint cost, and cost allocation functions.

We have thus shown the existence of conditions under which the government is strictly better off not sharing costs with the agents. In that case, the government should solicit bids from the agents, award the contract to the minimum bidder, and let the winning bidder bear all of the ex post cost of the contract.

IV. SUMMARY

McAfee and McMillan [1986] have shown that in a model in which opportunity costs can be directly observed, risk-sharing in the form of an incentive contract will be optimal even for risk-neutral bidders. We have shown that the presence of joint costs may destroy the optimality of incentive contracts for the case of risk-neutral bidders. In this case, observed costs and opportunity costs are not the same.

If a cost based incentive contract is to be employed, then observed cost must be clearly defined. When joint costs are present, this requires that joint costs be allocated between the output produced for the government and all other output. In our generalized model, we have seen that the presence of joint costs, and the need to allocate those costs, may cause a significant divergence between opportunity cost and observed cost. Cost sharing has a positive impact in that it stimulates bidding competition. However, that positive impact can be overwhelmed by cost sharing's inefficient cross-subsidization of non-government output. When this occurs, the government is strictly better off not sharing costs by using a fixed-price contract.

As MM observe, in practice most government contracts are fixed-price. They use their results to justify a recommendation that more incentive contracts should be used. However, our results may help to explain the use of fixed-price contracts because of the real world divergence between accounting costs and opportunity costs.

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FOOTNOTES

1. The product produced for the government does not have exactly the same characteristics as the product produced for the outside market. If it did, the government could purchase the good on the outside market. For example, the government may want to purchase toilet seats for submarines from manufacturers of toilet seats for the general market.

2. Note that if auditing costs exist but are known to be the same regardless of which agent wins the contract, then without loss of generality, we can assume that these costs are zero.

3. This cost can be interpreted as the opportunity cost of the contract; it includes both production cost and effort cost differentials over no government output.

4. See Coddington, pp. 43-44.

5. The requirement that {marginal revenue-marginal separable cost} be a positive constant is stronger than we need. As long as the difference is positive and decreasing slowly and condition (iii) is satisfied, the result of the Theorem still holds.

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